COMPUTATION OF PARAMETERS OF A MOVING UNIFORM FLUIDIZED BED OF POLYDISPERSE GRANULAR MATERIALS*

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Relations are presented describing the moving uniform fluidized bed and the computation procedure is proposed. The calculation program given in the language FORTRAN is verified by use of experimental data obtained in measurements with the moving fluidized bed of quartz sand and water.

This study is related to our earlier papers[1-3] where relations describing the moving uniform fluidized bed have been derived and experimentally verified. The moving fluidized bed is defined as such fluidized bed in which the resulting velocity of any component of the system of solid particles is non-zero. In the case of uniform moving fluidized bed, the porosity over the cross sectional area is constant and without fluctuations in time. Fluidization is considered to take place only in earth gravitational field and the flow of liquid and solid particles is assumed only in the vertical direction. Under these conditions it is possible to use for description only scalar quantities. As the positive direction is chosen the direction counter to the direction of gravitational acceleration. The cross-sectional area has the sign identical with the sign of the mean liquid velocity or velocity of solid particles.

The moving uniform fluidized bed of polydisperse granular material can be described[2,3] by the system of equations

$$w_i = w_f + u_c \left[1 - \frac{Y_i}{X_i(1 - \varepsilon)} \right], \quad [i = 1, 2, \ldots, k], \tag{1}$$

where

$$u_c \equiv \dot{V}_s / S, \tag{2}$$

and where

$$X_i \equiv V_{si} / V_s, \quad [i = 1, 2, \ldots, k], \tag{3}$$

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and where V_{si} and V_s are the volumes of the solid phase of the i-th component and of the whole polydisperse mixture in the fluidized bed, respectively.
There holds

$$\sum_{i=1}^{k} X_i = 1 . \tag{4}$$

The symbol Y_i denotes the volume fraction of the i-th component of the polydisperse material in the solid phase after the outlet from the fluidized bed. In steady state, the quantity Y_i also represents the relative volumetric flow rate of the i-th component of the polydisperse material in the solid phase of the moving fluidized bed and it is given by the relation

$$Y_i \equiv \dot{V}_{si}/\dot{V}_s , \quad [i = 1, 2, \ldots, k] , \tag{5}$$

where V_{si} and V_s are the volumetric flow rates of the i-th component and of the whole dispersion mixture of granular material, respectively. But of course there simultaneously holds

$$\sum_{i=1}^{k} Y_i = 1 . \tag{6}$$

By the system of Eqs (*1*) to (*6*) for all components of the polydisperse material together with the expansion relationships of components of the non-moving fluidized bed given *e.g.* by the relation

$$f(\mathrm{Re}_i, \mathrm{Ar}_i, \varepsilon) = 0 , \quad [i = 1, 2, \ldots, k] , \quad \mathrm{Re}_i \equiv d_i w_i/\nu \tag{7}$$

it is possible to describe the moving fluidized bed of polydisperse material. In Eq. (*7*) the quantity w_i is the superficial velocity of fluid in the non-moving fluidized bed ($u_c = 0$) of the isolated i-th component of solid phase. In studies[1-3] has been proved that values of the quantity w_i obtained for the non-moving fluidized bed can be applied in the description of the moving fluidized bed ($u_c \neq 0$) according to Eq. (*1*).

The parameters of the moving uniform fluidized bed of polydisperse material with the known composition can be thus calculated by simultaneous solution of the system of Eqs (*1*), (*4*) and (*7*). For this case it is necessary to solve $(2k + 1)$ equations. The same number of Eqs (*1*), (*6*) and (*7*) is available at the given composition of the solid phase of the fluidized bed.

In the majority of cases we know the composition of the inlet polydisperse material Y_i and its volumetric flow rate $\dot{V}_s$. It is usually of interest to obtain information on the cross-sectional area of the equipment S, on the superficial liquid velocity w_f in this equipment and on the porosity ε. In technical calculations it is usually possible to choose two of these quantities and one is the subject of calculation.

When the number of components of the polydisperse material is greater than two the computation is complicated and it is suitable to use the digital computer. The computation programme FLUSEP has been designed which is described in the following part of the study.

Description of the computation programme. In the first part of this study there is marked out the computation procedure for obtaining the basic information on the moving uniform fluidized bed of polydisperse material. From the numerical point of view the most difficult part of this procedure is the simultaneous solution of the system of non-linear algebraic Eqs (*1*), (*4*) and (*7*). For their solution the Newton–Raphson method has been used.

For the numerical computation, Eqs (*1*), (*4*) and (*7*) have been arranged into the form

$$w_t + u_c - Y_i u_c / X_i (1 - \varepsilon) - w_i = 0\,, \quad [i = 1, 2, \ldots, k]\,, \tag{1a}$$

$$\sum_{i=1}^{k} X_i - 1 = 0 \tag{4a}$$

and

$$A_i (w_i / w_{0i})^{B_i} - \varepsilon = 0\,, \quad [i = 1, 2, \ldots, k]\,. \tag{7a}$$

In the system of Eqs (*7a*), the quantities A_i and B_i are constants which are determined for each component of the polydisperse mixture experimentally or are obtained from the known relations[4].

Eqs (*1a*), (*4a*) and (*7a*) form the system of $(2k + 1)$ nonlinear algebraic equations which can be, in general, written in the form

$$\begin{aligned} F_1(x_1, x_2, \ldots, x_m) &= 0 \\ &\vdots \\ F_n(x_1, x_2, \ldots, x_m) &= 0\,. \end{aligned} \tag{8}$$

This system has a unique solution at the assumption that dim m = dim n which can be obtained by the variation method:

$$x^{(h+1)} = x^{(h)} + \lambda \, \Delta x^{(h)}\,. \tag{9}$$

The value $\Delta x^{(h)}$ is obtained from the relation

$$\Delta x^{(h)} = -\{\Gamma_F(x^{(h)})\}^{-1} F(x^{(h)})\,, \tag{10}$$

where the following expression

$$\Gamma_F(x^{(h)}) = \{\delta F_i / \delta x_j\}\,, \quad [i = 1, 2, \ldots, n]\,, \quad [j = 1, 2, \ldots, m] \tag{11}$$

is the Jacobi's matrix of partial derivatives and $F(x^{(h)})$ is the vector of residue of right hand sides of the system of Eqs (*10*) at iterations.

For improvement of convergence properties of the method especially at initial iterations are the increments $\Delta x^{(h)}$ multiplied by the coefficient $\lambda \in (0; 1)$ which is chosen so that it holds

$$\left\|F\left(x^{(h+1)}\right)\right\| < \left\|F\left(x^{(h)}\right)\right\|$$

while an arbitrary suitable standard can be used.

It is obvious from Eq. (*10*) that the solution cannot be obtained in case when the Jacobi's matrix is singular. The advantage of this method is in reliable convergence even at wrong selection of initial values of variables and the possibility of different initial specifications of the problem *i.e.* distribution into unknown and given variables. Partial disadvantage is the necessity to generate matrices of partial derivatives Γ_F in each iteration which is increasing the computation time and is increasing the requirements on the storage capacity of the computer. This disadvantage becomes more obvious with larger systems of equations (200 and more equations).

The programme FLUSEP is written in the FORTRAN language. It consists of the main programme and seven subprogrammes.

The main programme is used for control of the computation, print of the output values and generation of constants A_i and B_i in Eq. (*7a*).

The inlet values characterizing the given system are introduced by the subroutine CTENI, the initial system of variables is composed and the distribution of variables into unknowns and given quantities is introduced. The system of variables is arranged in the following way: Thus we have in general $(2k + 3)$ variables for $(2k + 1)$ equations. Therefore two variables must be given so that the problem could be solved. In this vector are not included for simplificatition the variables Y_i and w_{0i} (for $i = 1, 2, \ldots, k$). They cannot be the aim of the computation and they must be always given.

The subroutine FLUIDS is used for generation of individual terms of the matrix Γ_F. Derivatives are generated line after line which are fixed by use of subroutine SESPD and ZAMENA into the arrangement suitable for computation.

So arranged system is then solved by use of standard subroutines DIAKUB and GAUSD which are described in detail in literature[5]. The procedure of computation by use of routine FLUSEP is schematically given in the form of a block diagram in Fig. 1.

Experimental and computed porosities of the uniform moving fluidized bed of a six component mixture of the quartz sand were compared. The arrangement of fluidized bed was co-current with positive direction of motion of both liquid (water) and granular material (sand).

TABLE I

Experimental and Computed Porosities ε of the Co-currently Moving Fluidized Bed of Polydisperse Material

Run No	w_f mm/s	u_c mm/s	ε	
			experimental	computed
1	32·4	4·18	0·597	0·593
2	62·9	4·29	0·749	0·750
3	97·7	4·61	0·844	0·861
4	33·2	2·17	0·614	0·595
5	58·4	2·42	0·743	0·733
6	93·5	2·20	0·846	0·873

Start
Reading and printout of input parameters and initial estimated values of all variables
Reading and printout of constants A and B in the expansion relation (7a) for all components and input data in the range of their validity
Composition of vector. variables and their separation to given and unknown
1
Generation of constants A and B for all components
Composition of the matrix of partial derivatives and of right hand sides; arrangement for computation
Computation and printout of sum of squared residua (SSR) of right hand sides of equations
Starting with the first iteration comparison of the new SSR with the previous one
new SSR is smaller
yes
2
not
Increments $\Delta x^{(h)}$ are halved
1

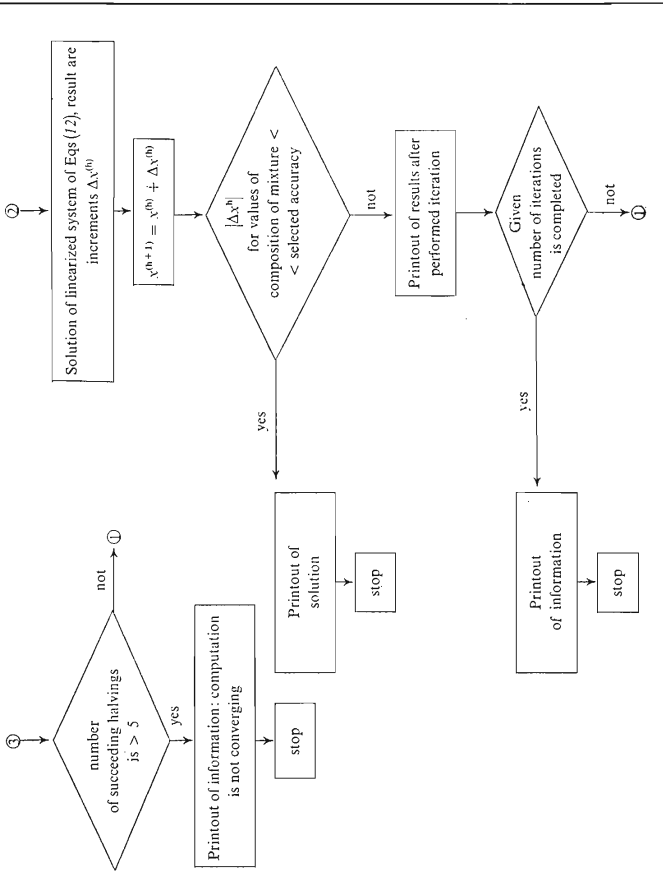

FIG. 1
Block Diagram

The mixture had the composition (equivalent diameter d_i and volume fraction Y_i):

0·476 mm—0·0403 ; 0·611 mm—0·0984 ; 0·724 mm—0·2357 ;
0·876 mm—0·2891 ; 1·174 mm—0·2777 ; 1·404 mm—0·0588 .

Characteristic values of properties of individual components have been given in our recent studies[1,3]. The expansion relationships (*7*) were for each component determined experimentally[6]. The experimental and computed values are given in Table I. From comparison of computed and experimentally determined values of porosities can be observed their very good agreement which corresponds to the results[1,2] given earlier. The maximum deviation of the computed and experimental porosity is 3%.

The developed programme FLUSEP can be used for determination of the maximum flow rate of granular material through the already operated equipment or for design of needed dimensions of the equipment for the given flow rate of solid particles. For monodisperse particles these problems have also been solved by other authors[3,7,8]. The programme can be also applied both for control of a process and for its optimization.

From the obtained results it is possible to conclude that the proposed procedure of computation of parameters of the moving fluidized bed is reliable. The maximum number of iterations in the computation was from 10 to 15.

LIST OF SYMBOLS

A_i constant in Eq. (*7a*)
$Ar_i \equiv g d_i^3 (\varrho_{si} - \varrho_f)/\nu^2 \varrho_f$ Archimedes number
B_i constant in Eq. (*7a*)
d_i equivalent diameter of particle of the *i*-th component of polydisperse mixture
F arbitrary function
g gravitational acceleration
(h) number of iteration
i ordinal number of component
k total number of components of the polydisperse mixture
$Re_i \equiv d_i w_i/\nu$ Reynolds number
S cross sectional area
u_c superficial velocity of a compact bed of particles ($\varepsilon = 0$), defined by Eq. (*2*)
$\dot{V}_s$ volumetric flow rate of the solid phase
w_i relative superficial velocity of fluid with respect to the *i*-th component of polydisperse mixture of particles
w_f superficial velocity of fluid
X_i volume fraction of the *i*-th component of polydisperse material in solid phase of the fluidized bed
x parameter of Eq. (*10*)
Y_i relative flow rate of the *i*-th component defined by Eq. (*5*)
Γ_F Jacobi's matrix of partial derivatives defined by Eq. (*13*)
ε porosity of bed
ϱ_f density of fluid
ϱ_s density of particles
ν kinematic viscosity of fluidizing fluid

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